Strutture dati per la Rappresentazione di Dati Vettoriali

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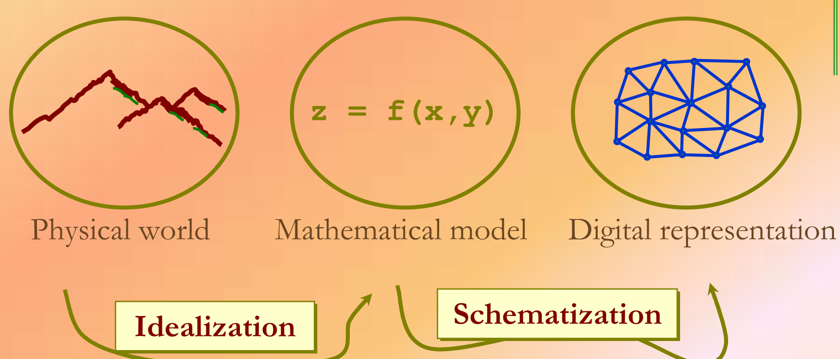
GE-IMATI-CNR

Geometric modelling



Basics on algebraic topology

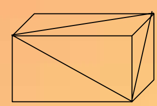
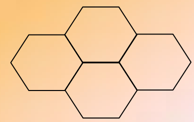
- Algebraic topology associates algebraic invariants to each space so that two space are homeomorphic if they have same invariants
- Approach: to break a topological space into simple pieces that are easier to study (e.g. to decompose a polyedron into faces, edges, vertices or a surface into triangles)

Basics on algebraic topology

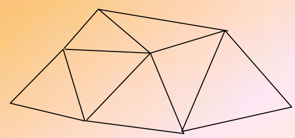
- A combinatorial structure is generated decomposing the topological space
- Basic elements of the decomposition are cells or simplices that are characterized by
  - combinatorial aspects: relations among the cells of the complex
  - geometric aspects: which concern their embedding in the Euclidean space

Examples

- cells complexes
  - a geographic map (which is formed by points, lines and regions)
  - the decomposition of a polyhedron into faces

- simplicial complexes
  - triangle meshes



Abstract simplicial complex

- Let V a set. An abstract simplicial complex $\Sigma_A \subseteq \wp(V)$ is a set satisfying the following conditions:
  - $\varnothing \in \Sigma_A$,
  - $\forall v \in V$, $\{v\} \in \Sigma_A$,
  - $\Sigma \tau \subseteq \sigma \in \Sigma_A$, then $\tau \in \Sigma_A$
- e.g. $\{\{a,b,c\},\{c,d\}, \{e\}\}$
- order= the maximum cardinality of the simplices
- any abstract simplicial complex of order d has a geometric realization in $\Re^{2d+1}$

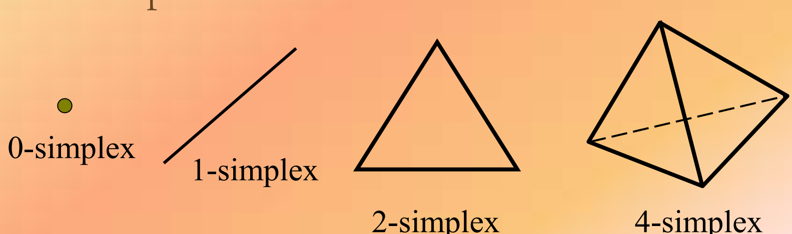
Geometric realization

- Let V_σ be a set of $d+1$ linearly independent points in the n-dimensional Euclidean space with $d \leq n$
- A d-simplex generated by V_σ is the subset of $\Re^n$ given by the points x that can be expressed as the convex combination of the points of V_σ

$$x = \sum_{i=0}^{d} l_i v_i \quad \text{with } l_i \in [0,1] \text{ and } \sum_{i=0}^{d} l_i = 1$$

Examples

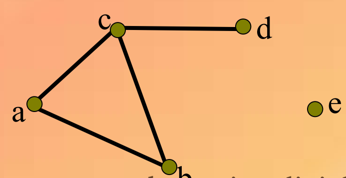
- d-simplices



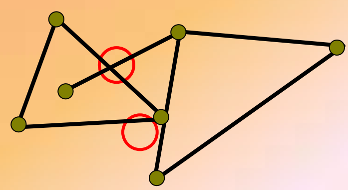
- Alternatively d-simplices may be defined as the convex hull of the points in V_σ

Simplicial complex

- realization of the simplex{{a,b,c},{c,d}}:



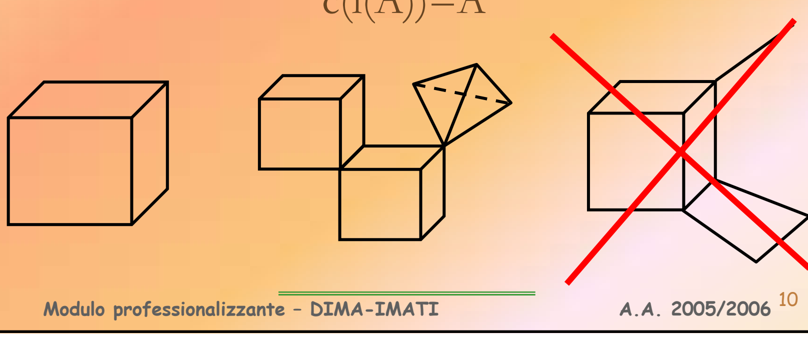
- a set of simplices may not be a simplicial complex:



Regular sets

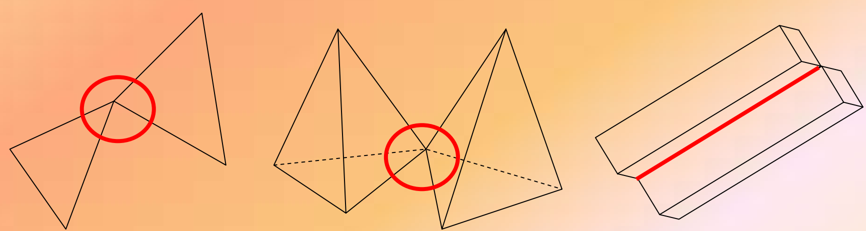
- A connected, compact set A⊆ℜ^n is a *regular set* (*r-set*) if A equals the closure in ℜ^n of its interior:

$$c(i(A))=A$$



Manifolds

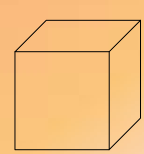
- In our case, when modelling a discrete space only r-sets with boundary 2-manifold will be considered



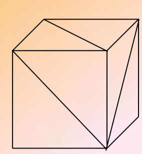
Euler characteristic

$$\chi(S)=v-e+f=2(s-g)+h$$

- v=#vertices
- e=#edges
- f=#faces
- S=#connected components
- g=genus of the surface
- h=#boundary components



v=8, e=12, f=6



v=8, e=18, f=12

BUDDAH: v=32328, f=64676, e=97014, holes=6, χ =-10

RABBIT: v=8500, f=16996, e=25494, holes=0, χ =2

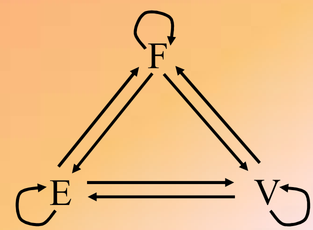
TORUS: v=256, f=512, e=768, holes=1, χ =0

Representation schemes

- are relations which associate to a mathematical model a representation
- a representation must be
  - non ambiguous
  - unique
- boundary schemes represent a shape by describing its boundary

Data structures

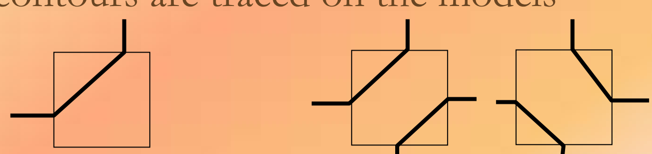
- topological relations of a boundary representation



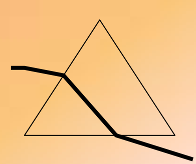
- time constant: EV, EE, EF
- time varying: VV, VE, VF, FV, FE, FF

Contour lines on square and triangle meshes

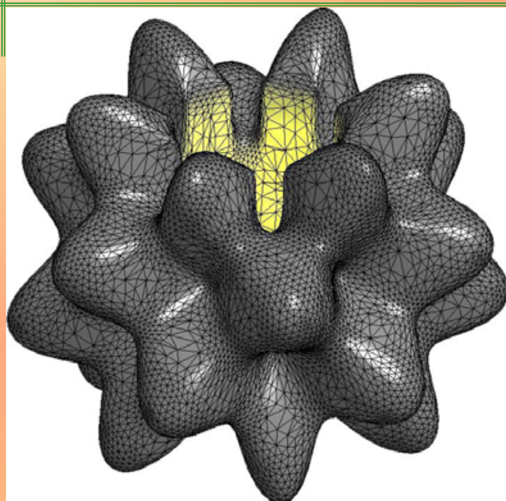
- square cells may have some ambiguities when contours are traced on the models

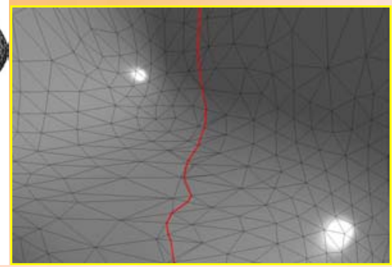


- triangle meshes overcome the ambiguity problem

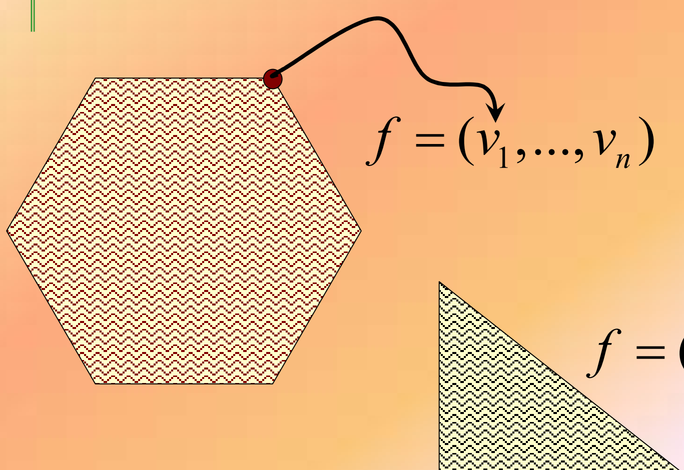


Coding and moving on simplicial complexes





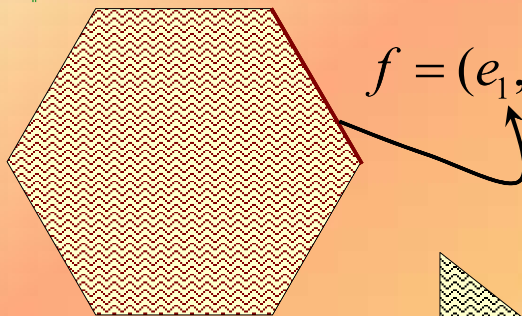
Face-Vertex: associates to a face the lits of its vertices



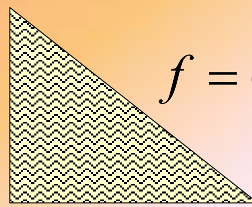
$$f = (v_1, \dots, v_n)$$

$$f = (v_1, v_2, v_3)$$

Face-Edge: associates to a face the list of its edges



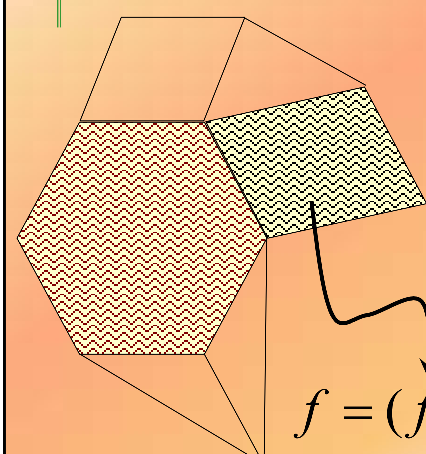
$$f = (e_1, \ldots, e_n)$$



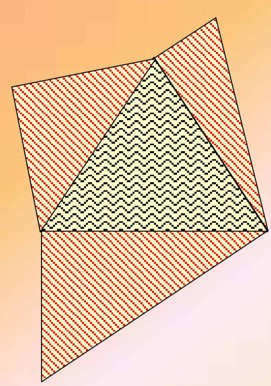
$$f = (e_1, e_2, e_3)$$

Face-Face: associates to a face the list of its adjacent faces

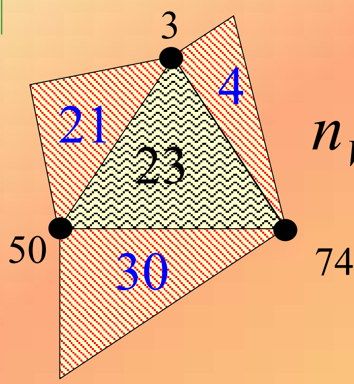
$$f = (f_1, f_2, f_3)$$



$$f = (f_1, \ldots, f_n)$$



File Format & Input data structure



File (VER): float/double

n_V

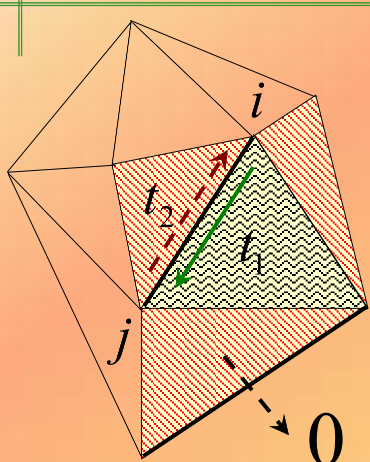
Row 3| ……………….
Row 50| 0.12 3.3 -0.34
Row 74 ……………….

File (TRI): integers

n_T

Row 22| ……………….
Row 23| 3 50 74 21 30 4
Row 24| ……………….

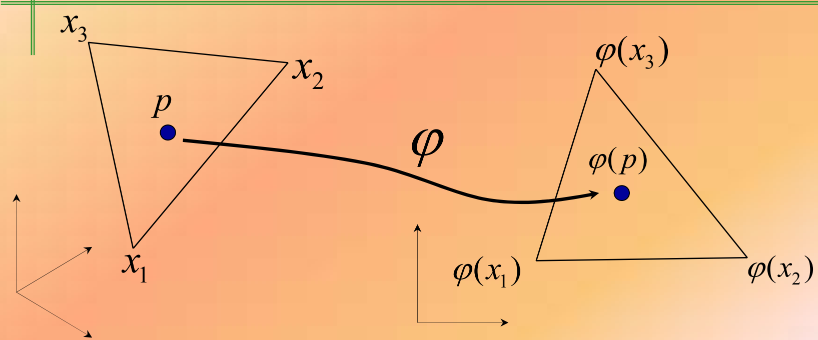
How to extract the face-face relation?



$$T \in M_{n_V, n_V}(\mathbb{Z})$$

$$T_{ij} = t_1, T_{ji} = t_2$$

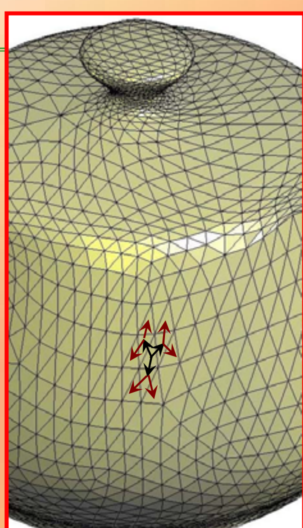
Barycentric coordinates



$$p = \lambda_1 x_1 + \lambda_2 x_2 + \lambda_3 x_3 \Leftrightarrow \varphi(p) = \lambda_1 \varphi(x_1) + \lambda_2 \varphi(x_2) + \lambda_3 \varphi(x_3)$$

$$\lambda_i \geq 0, i = 1,2,3, \lambda_1 + \lambda_2 + \lambda_3 = 1$$

How to count the number of sheels?



<u>Idea:</u> use recursively the TT relation while marking triangles as visited and until the whole set of faces has been processed.

<u>Computational cost</u>: linear in the number of input triangles.

Esercitazione – Parte A: Superfici Triangolate

- 1. Scrivere una funzione di caricamento di un file OFF
  [mat_ver,mat_tri]=load_off(file_name);

- 2. Usando la relazione FF, calcolare il numero di componenti connessi di una superficie triangolata arbitraria (con o senza bordo). Per ciascuna componente connessa (shell) estrarne:
  - le (eventuali) componenti di bordo;
  - il genere;
  - l'area.

  shell=extract_shells(mat_tri);