3D Shape Description and Matching Based on Properties of Real Functions

Comparison Methodologies

Eurographics 2007 Tutorial T12

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Evaluating the matching characteristics

- **Properties** of the similarity measure
- **Robustness** of the similarity measure
  - *Low* variation of the measure wrt *small* variations of the shape descriptor
- **Type of comparison**
  - global and/or partial matching
- **Type of information** taken into account
  - geometrical, topological, structural
- **Computational complexity** of the matching algorithm
- **Application context**

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Properties of similarity measures

- Let **S** be the set of shape descriptors, the distance measure **d** between two shapes descriptors is defined as:

$$d : S \times S \rightarrow \Re$$

- Properties:
  - $d(x,x) = 0$ (self identity)
  - $d(x,y) > 0,\ x \neq y$ (positivity)
  - $d(x,y) = d(y,x)$ (symmetry)
  - $d(x,z) \leq d(x,y) + d(y,z)$ (triangular inequality)
  - $d(x,z) \leq \max\{d(x,y), d(y,z)\}$ (strong t. i.)

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Properties of similarity measures

- The measure properties are grouped as in the following:
  - **semi-metric**: self-identity, positivity, symmetry
  - **pseudo-metric**: self-identity, symmetry, triangular inequality
  - **metric**: a pseudo-metric that satisfies the positivity
  - **ultra-metric**: a metric satisfying strong triangular inequality
- The perceptual space can be approximated by the metric properties ? [Tve77,SJ99]
  - symmetry and triangular inequality should not holds for partial matching
- Metrics can be used for indexing purposes

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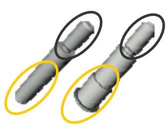
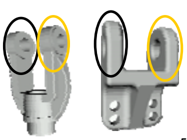
Type of comparison

- Global Matching:
  - Overall shape comparison
  - Real number representing the similarity estimation between the two objects

- Sub-Part Correspondence:
  - Real number as similarity estimation
  - Mapping among similar sub-parts



- Partial Matching:
  - Real number as similarity estimation
  - Similar sub-parts between objects having different overall shape



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Type of information taken into account

- according to the type of information stored and the way it is coded in the descriptor, the measure of similarity may take into account:
  - geometric information
  - topologic information
  - structural information

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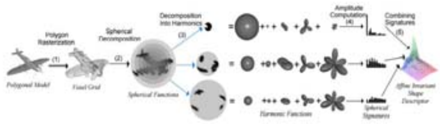
Comparison methodologies

- for descriptors represented by matrices and vectors
  - Spherical Harmonic representation [KFR03]
  - Shape DNA [RWP06]
  - Bending Invariant Surface Signatures[EK03,BBK06]
  - Spectral Embedding [JZ07]
  - Pose-oblivious shape signature [GSCO07]
  - Salient geometric features [GCO06]
- for descriptors represented by graphs
  - Multiresolution Reeb Graphs [HSKK01]
  - Extended Reeb Graphs [BMSF06]
- for descriptors represented by formal series
  - Size functions [dFL], [dAFL05]
  - Barcodes and persistence diagrams [CZCG05]

Spherical Harmonic representation [KFR03]

- Represent a function *f* defined on the sphere through its spherical harmonics and consider the vector of energies (i.e. frequency norms)
- Extension to voxel description:
  - Restrict the voxel grid to a collection of concentric spheres
  - Represent each spherical restriction in terms of the energy of its frequency decomposition, thus obtaining a 1D descriptor
  - The final descriptor resulting from the analysis of spheres with different radii is a 2D grid indexed by radius and frequency
- 2D descriptors are compared by using the L_2 norm



Spherical Harmonic representation: matching characteristics [KFR03]

- **Properties** of the similarity measure
  - metric
- **Robusteness** of the similarity measure
  - induced by the properties of metrics
- **Type of comparison**
  - global matching
- **Type of information** taken into account:
  - geometric information
- **Computational complexity** of the matching algorithm
  - linear in the number of entries stored in the 2D array
- **Application context**
  - retrieval of 3D objects, not suitable for articulated objects

Shape DNA [RWP06]

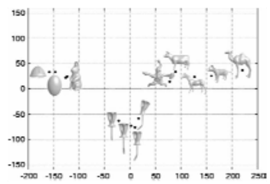
- Shape DNA signatures are *m*-dimensional feature vectors that can be compared using any metric between vectors, e.g. the Euclidean *p*-norm

$$d_p(u,v)=\left(\sum_{i=1}^{m}|u_i-v_i|^p\right)^{\frac{1}{p}}$$

the Hausdorff distance, the Pearson correlation distance
- according to empirical evidence, d_2 yields good results while being easy to compute

Shape DNA [RWP06]

- Matching results on a small database of meshes, including different classes of deformed models, show a nice clustering of objects



- Other experiments on collections of grey-scale and colour images [RWP07]
- Medical applications on brain surfaces [NRW07], using statistical methods to distinguish populations; extention to 3D brain data

Shape DNA: matching characteristics [RWP06]

- **Properties** of the induced similarity measure
  - metric (using the Euclidean *p*-norm)
- **Robustness** of the similarity measure:
  - induced by the robustness of metrics
- **Type of comparison**: global matching
- **Type of compared information**
  - the descriptor stores geometric and topological information, but it is difficult to control them in a differentiated manner in the definition of the measure
- **Computational complexity** of the matching algorithm
  - p-norms are linear in the number of vertices of the model
- **Application context**:
  - medical applications, suitable for articulated objects

Bending Invariant Surface Signatures [EK03]

- Given the surface signatures, any algorithm to evaluate the similarity of rigid objects can be involved in the comparison step
- Example: Compute the vectors of the first few moments of the surfaces and compute their Euclidean distance

Bending Invariant Surface Signatures [EK03]

- **Properties** of the induced similarity measure
  - Depends on the matching method used
- **Robustness** of the similarity measure
  - Depends on the matching method used
- **Type of comparison**
  - Global or partial matching depending on the matching method used
- **Type of compared information**
  - Depends on the matching method used
- **Computational complexity** of the matching algorithm
  - Depends on the matching method used
- **Application context**
  - face recognition

Spectral Embedding [JZ07]

- Compare shapes by computing existing shape descriptors (Light Field, Spherical Harmonics) on spectral embeddings
- Use the vectors of normalized eigenvalues and define:

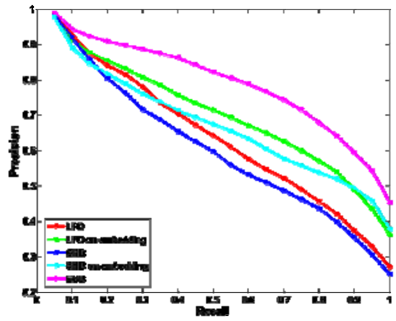
$$D_{EVD}(Q,S)=\frac{1}{2}\sum_{i=1}^{m}\frac{\left[\left|\lambda_i^Q\right|^{\frac{1}{2}}-\left|\lambda_i^S\right|^{\frac{1}{2}}\right]^2}{\left|\lambda_i^Q\right|^{\frac{1}{2}}+\left|\lambda_i^S\right|^{\frac{1}{2}}}$$

- Compute a correspondence cost derived from the correspondence between the vertices of the two shapes (possibly after a first filter using EVD)

$$D_{CCD}(Q,S)=\sum_{p\in Q}\left\|V_Q(p)-V_S(match(p))\right\|$$

Spectral Embedding [JZ07]

Precision-Recall plot for McGill database



Spectral Embedding: matching characteristics [JZ07]

- **Properties** of the induced similarity measure
  - $D_{EVD}(Q,S)=\frac{1}{2}D(f,g);$
  - $D(f,g)=\chi^2=\int\frac{(f-g)^2}{f+g},\quad f=\left|\lambda_i^Q\right|^{\frac{1}{2}},\ g=\left|\lambda_i^S\right|^{\frac{1}{2}}$
  - X^2 is a semi-metrics if *f* and *g* are positives
- **Robustness** of the similarity measure
  - induced by the robustness of metrics

Spectral Embedding: matching characteristics [JZ07]

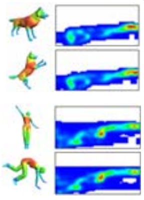
- **Type of comparison**
  - Global matching
- **Type of compared information**
  - geometric and topological information
- **Computational complexity** of the matching algorithm
  - D_{EVD} is linear in the number of vertices of the embedded model
- **Application context**
  - suitable for articulated objects

Pose-oblivious shape signature[GCO06]

- The pose oblivious is a 2D histogram that combines local diameter function and centricity function both defined on the boundary surface of the 3D shape.
- Matching:
  - correlation coefficient:

$$R(P,Q) = \frac{N\sum p_i q_i - \sum p_i \sum q_i}{\sqrt{(N\sum p_i^2 - (\sum p_i)^2)(N\sum q_i^2 - (\sum q_i)^2)}}$$

  - $\chi^2 = D(f,g) = \int \frac{(f-g)^2}{f+g}$



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Pose-oblivious shape signature : matching characteristics [GSCO07]

- **Properties** of the similarity measure
  - X^2 is a semi-metrics if *f* and *g* are positives
  - correlation coefficient is a semi-metric
- **Robustness** of the similarity measure
  - induced by the properties of measures
- **Type of comparison**
  - global matching
- **Type of information** taken into account
  - the descriptor stores geometric information
- **Computational complexity** of the matching algorithm
  - linear in the number of entries stored in the 2D array
- **Application context**
  - retrieval of 3D objects, suitable for rticulated objects

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Salient geometric features [GCO06]

- Each salient feature is associated with a vector index (a signature) and inserted into a geometric hash table
- Given a query object, its salient feature are extracted and used to query the database for a list of matching features
- The returned features identify the models having larger number of matches.

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Salient geometric features [GCO06]

- The vector index used in the hash table encode the following information:
  - area of the salient feature
  - curvature of the salient feature
  - number of local minimum(s) or maximum(s) curvatures in the salient feature
  - the curvature variance in the salient feature
- The similarity between objects is given by the number of correspondence among the salient features

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Salient geometric features : matching characteristics [GCO06]

- **Properties** of the similarity measure
  - similarity measure is not proposed by authors
- **Type of comparison**
  - Sub-part correspondence and partial matching.
- **Type of information** taken into account:
  - geometric information
- **Computational complexity** of the matching algorithm
  - depends on the geometric hashing used
- **Application context**
  - retrieval of 3D objects, object alignement

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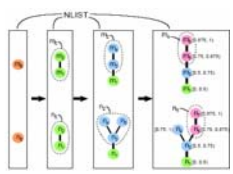
Multiresolution Reeb Graph [HSKK01]

- Similarity between two nodes *P*,*Q* is weighted on their attributes:

$$sim(P,Q) = \alpha\,|A(P) - A(Q)| + (1-\alpha)\,|L(P) - L(Q)|\,,\ 0 < \alpha < 1$$

- Nodes with maximal similarity are paired if:
  - Share the same range of *f*
  - Parent nodes are matched
  - Belong to graph paths already matched
- The distance between two MRGs is the sum of all node similarities



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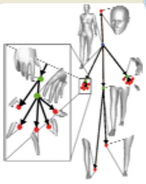
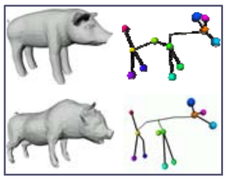
Multiresolution Reeb Graph: matching characteristics [HSKK01]

- **Properties** of the induced similarity measure
  - metric
- **Robustness** of the similarity measure
  - stability properties of metric
- **Type of comparison**
  - global matching (suitable also for sub-part correspondence and partial matching)
- **Type of compared information**
  - structural and geometric information
- **Computational complexity** of the matching algorithm
  - $O(M \cdot (M+N))$ where M and N is the number of nodes of the two multiresolution graphs
- **Application context**
  - Retrieval of free form objects

Extended Reeb Graphs [BMSF06]

- Two ERGs are compared using a graph-matching approach based on the "best common subgraph" detection
- Also sub-part correspondences are recognized
- Heuristics are used to improve
  - Quality of the results
  - Computational time





Extended Reeb Graphs [BMSF06]

- Given G_1 and G_2, two direct, acyclic and attributed graphs:
  - the distance d between two nodes $v_1 \in G_1$ and $v_2 \in G_2$ is

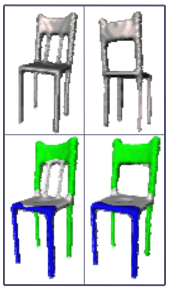
$$d(v_1, v_2) = \frac{w_1 G_S + w_2 St_S + w_3 Sz_S}{3} \qquad w_i \in [0,1] \qquad \sum w_i = 1$$

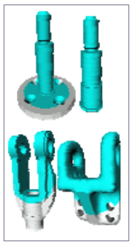
  - the distance $D(G_1, G_2)$ depends both on the geometry and the structure of the objects:

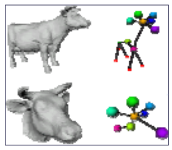
$$D(\mathcal{G}1, \mathcal{G}2) = 1 - \frac{\sum_{v \in \mathcal{G}} (1 - d(\psi_1(v), \psi_2(v)))}{max(|\mathcal{G}1|, |\mathcal{G}2|)}$$

Extended Reeb Graphs [BMSF06]

- Some examples of sub-part correspondence and partial matching







Extended Reeb Graphs: matching characteristics [BMSF06]

- **Properties** of the Induced similarity measure
  - semi-metric
- **Robustness** of the similarity measure
  - Stability properties of semi-metrics
- **Type of comparison**
  - global matching, sub-part correspondence and partial matching
- **Type of compared information**
  - structural and geometric information
- **Computational complexity** of the matching algorithm
  - $O(n^3)$ where n is $\max\{|G_1|, |G_2|\}$
- **Application context**
  - free form objects and CAD models

Matching distance between 1-dimensional size functions [dAFL06]

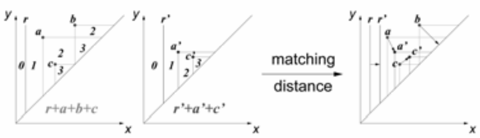
- Two size functions ℓ_1, ℓ_2, with associated formal series C_1 and C_2, can be compared by measuring the reciprocal distances of cornerpoints and cornerlines

$$\delta((x,y),(x',y')) = \min\left\{\max\{|x-x'|, |y-y'|\}, \max\left\{\frac{y-x}{2}, \frac{y'-x'}{2}\right\}\right\}$$

and choosing the matching which minimizes the maximum of these distances

$$d_{match}(\ell_1, \ell_2) = \min_{\sigma} \max_{p \in C_1} \delta(p, \sigma(p))$$

when σ varies among the bijections between C_1 and C_2



Matching distance between 1-dimensional size functions [dAFL06]

- Matching Stability Theorem:
  The matching distance satisfy the following stability condition:

$$\max_{P\in\mathcal{M}} |\varphi(P)-\psi(P)| \le \epsilon \Rightarrow d_{match}(\ell_{(\mathcal{M},\varphi)}, \ell_{(\mathcal{M},\psi)}) \le \epsilon.$$

- Lower bound for the natural pseudo-distance:
  Let λ be the value of matching distance between the two size functions $\ell_{(M,\varphi)}$ e $\ell_{(N,\psi)}$. Then

$$d((M,\varphi),(N,\psi)) \ge \lambda.$$

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Matching distance between multidimensional size functions [BCF*07]

- On each leaf of a particular foliation of their domain, multidimensional size functions coincide with a particular 1-dimensional size function
- the induced 1D matching distance on each leaf of the foliation is stable wrt small changes of the leaves;
- a multidimensional matching distance can be defined

$$D_{match}\left(\ell_{(\mathcal{M},\vec\varphi)}, \ell_{(\mathcal{N},\vec\psi)}\right) = \sup_{(\vec l,\vec b)} \min_{i=1,\dots,k} l_i \cdot d_{match}\left(\ell_{(\mathcal{M},F^{\vec\varphi}_{(\vec l,\vec b)})}, \ell_{(\mathcal{N},F^{\vec\psi}_{(\vec l,\vec b)})}\right)$$

- theorems about the stability of the matching distance and the lower bound for the natural pseudo distance can be stated also in the case k>1

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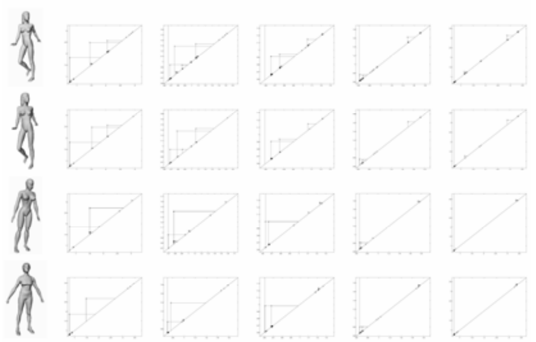
Multidimensional Size Functions [BCF*07]



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Size Functions: matching characteristics [dAFL06,BCF*07]

- **Properties** of the induced similarity measure
  - the matching distance is a metric
  - it provides a lower bound for the natural pseudo-distance
- **Robustness** of the similarity measure
  - stability theorem for the matching distance
- **Type of comparison**
  - global matching
- **Type of compared information**
  - geometric-topological
- **Computational complexity** of the matching algorithm
  - O($n^{2.5}$), where n is the number of cornerpoints taken into account
- **Application context**
  - Medical images, trademarks recognition, 3D retrieval

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Barcodes [CZCG05]

- I, J intervals in a barcode, $\delta(I,J) = |I \cup J - I \cap J|$
- A matching between barcodes S_1, S_2 is the set
  $$M(S_1,S_2) \subseteq S_1 \times S_2 = \{(I,J)\ s.t.\ I \in S_1,\ J \in S_2\}$$
  s.t. any interval in S_1 or S_2 occurs in at most one pair (I,J)
- Distance between S_1, S_2 relative to M
  $$D_M(S_1,S_2) = \sum_{(I,J)\in M} \delta(I,J) + \sum_{L\in N} |L|$$
  with N the set of non matched intervals

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Barcodes [CZCG05]

- Barcode pseudo-metric:
  $$D(S_1,S_2) = \min_M D_M(S_1,S_2)$$
- Minimizing D_M is equivalent to maximizing the similarity
  $$S_M(S_1,S_2) = \frac{1}{2}\left(\sum_{S_1}|I| + \sum_{S_2}|J| - D_M(S_1,S_2)\right)$$
- Recasting the problem as a graph problem, such minimization is equivalent to the well known maximum weight bipartite matching problem

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Barcodes [CZCG05]

- Examples on mathematical surfaces
- Classification results on a set of 80 hand-drawn copies of letters

Persistence Diagrams [CSEH07]

- Describing $\mathscr{P}$– intervals as point sets in the extended plane, i.e. by persistence diagrams, the Bottleneck Stability Theorem has been proved
- Under conditions on the space and the functions f, g, it holds that the Bottleneck distance between persistence diagrams $D(f)$, $D(g)$ satisfies

$$d_B\big(D(f), D(g)\big) \leq \|f-g\|_\infty$$

where d_B is defined as

$$d_B(X,Y) = \inf_\gamma \sup_x \|x-\gamma(x)\|_\infty$$

with X, Y multisets of points, $x \in X$, $y \in Y$ range over all points and γ ranges over all bijections from X to Y

Barcodes and Persistence Diagrams [CSEH07]

- In terms of persistence diagrams, the distance defined for barcodes can be written

$$d(D_1, D_2) = \inf_\gamma \sum_p \|p-\gamma(p)\|_1$$

with γ ranging in the set of bijections between D_1 and D_2, but this distance does not guarantee the stability property proven for persistence diagrams under the Bottleneck distance
- Under certain assumptions, the Barcode Theorem holds, guaranteeing the stability property under the Bottleneck distance

Barcodes and persistence diagrams: matching characteristics [CZCG05,CSEH07]

- **Properties** of the induced similarity measure
  - pseudo-metric between barcodes
  - metric between persistence diagrams
- **Robustness** of the similarity measure
  - stability theorems for barcodes and persistence diagrams under the Bottleneck distance
- **Type of comparison**
  - global matching
- **Type of compared information**
  - geometric-topological
- **Computational complexity** of the matching algorithm
  - for the pseudo-metric between barcodes, it depends on the algorithm used to minimize D(s_1, s_2)
- **Application context:** 3D and curve PCD comparison

remarks

- Most of the discussed methods use descriptors encoded as matrices or vectors
- Very few results on robustness of the similarity measure
- Very few methods deals with partial matching

QUESTIONS?