The Hitchhiker's Guide to the Galaxy of Mathematical Tools for Shape Analysis

SIGGRAPH 2012 Course Notes

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Bianca Falcidieno

Daniela Giorgi

outline

- ✓ motivation
- ✓ mathematics and shape analysis challenges *(2:05– 2:15)*
  - shape properties and invariants
  - similarity between shapes
- ✓ mathematical guide (Part 1) *(2:15-3:10)*
  - topological spaces, functions, manifolds
  - metric spaces, isometries, curvature, geodesics
  - Gromov-Hausdorff distance
- ✓ concepts in action (Part 1) *(3:10-3:35)*
  - symmetry detection
  - surface correspondence
  - shape characterization

outline

- ✓ mathematical guide (Part 2) *(3:45-4:30)*
  - basics on algebraic topology and homology
  - Morse theory
  - natural pseudo-distance
- ✓ concepts in action (Part 2) *(4:30-5:00)*
  - persistent topology
  - Reeb graphs
- ✓ conclusions *(5:00-5:15)*

where are we now?

- ✓ technology today
  - plenty of 3D acquisition techniques
  - hardware for visualizing 3D on the desktop
  - computer networks: fast connections, low cost
  - 3D printers: not only mock-ups but even end products

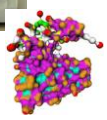
rendering, acquiring, transmitting, "materializing" 3D content is now feasible in specialized as well as unspecialized contexts

3D media

- professionals
  - Product Modeling & Design
  - Cultural Heritage
  - Gaming
  - Spatial Data
  - Simulation
  - Medicine
  - Bioinformatics
  - Architecture
  - Archaeology
- non professionals
  - 3D social networking
  - fabbing
  - ...






… how to analyse, describe, process, organize, navigate, filter, share, re-use and re-purpose, this large amount of complex content ?

reasoning about shape, similarity, semantics

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SIGGRAPH 2012 *Course Notes*

Mathematics and shape analysis challenges

shape and geometry

- "… all the geometrical information that remains when location, scale, and rotational effects are filtered out from an object" [Kendall 1977]

shape and *similarity*

- "…the form of something by which it can be seen (or felt) different by something else" [Longman Dictionary of Contemporary English]

that sounds nice but… what do "similar" and "different" mean?

shape, similarity & the observer

- things possess a shape for the observer, in whose mind the association between the perception and the existing conceptual models takes place [Koenderink 1990]

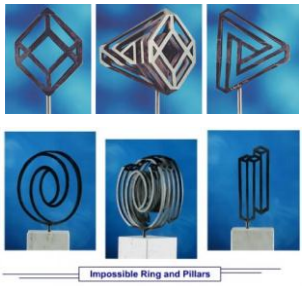
understanding, reasoning, similarity is a cognitive process, depending on the observer and the context

shape, similarity & the observer

- **things** possess a **shape** for the **observer**, in whose mind the association between the **perception** and the existing **conceptual** models takes place [Koenderink 1990]

understanding, reasoning, similarity is a cognitive process, depending on the observer and the context

shape and view points



shape and similarity

objects and similarity

geometric congruence

structural equivalence

functional equivalence

semantic equivalence

objects and similarities

geometric congruence

structural equivalence

functional equivalence

semantic equivalence

intuition vs mathematics

- congruence
  - two objects are congruent if one can be transformed into the other by rigid movements (translation, rotation, reflection – **not scaling**)

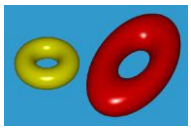
intuition vs mathematics

- similarity
  - two geometrical objects are called *similar* if one can be obtained by the other by uniform stretching . Formally, a *similarity* of a Euclidean space S is a function $f: S \rightarrow S$ that multiplies all distances by the same positive scalar r, so that:
    $d(f(x), f(y)) = rd(x, y), \forall x, y \in S$

intuition vs mathematics

- affinity
  - it preserves collinearity, i.e. maps parallel lines into parallel lines and preserve ratios of distances along parallel lines
  - it is equivalent to a linear transformation followed by a translation

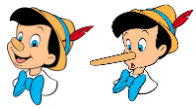
transformations and similarities



affine transformation

image from http://cse.taylor.edu/~btoll/s99/424/res/mtu/Notes/geometry/geo-tran.htm

isometric transformation



"locally-affine" transformation

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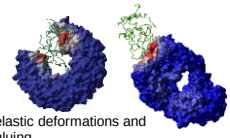


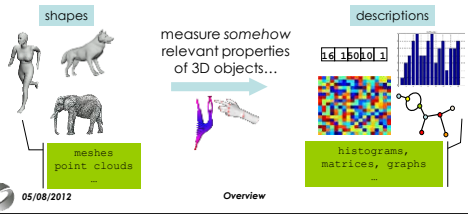
elastic deformations and gluing

mathematics: shape description and similarity

- similar shapes with **respect to what**?
  - shape descriptions, to code the aspects of shapes to be taken into account and manage the complexity of the problem
- **similarity** in what sense ?
  - transformations among the shapes that we consider irrelevant to the assessment of the similarity
    - invariants or properties

shape and description

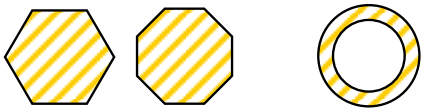
- shape descriptions reduce the complexity of the representation; their choice depends on
  - **type** of shapes and their variability/complexity
  - **invariants** or properties



shape descriptions

- in general, a description could be just a set of numbers…

example



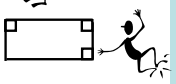
shape descriptions

- ✓ different shapes should have different descriptions
  - different enough to discriminate among shapes
- ✓ a shape may not be entirely reconstructed from its description

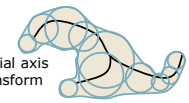
example

edges

edge length and angle



medial axis transform

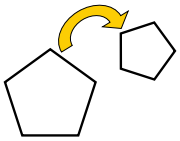


shape descriptions

- ✓ invariance
- ✓ uniqueness
- ✓ stability to noise
- ✓ sensitivity to global/local features

example

edges



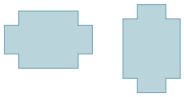
invariance

- ✓ invariance = the descriptor does not change for a given object under a class of transformations
- ✓ a property P is invariant to a transformation T applied to an object O iff

$$P(T(O)) = P(O)$$

example

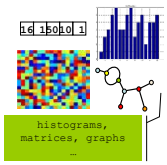
boundary length



shape descriptions and similarity

- ✓ *similarity* in what sense ?
  - defining appropriate *similarity measures* between shape descriptions

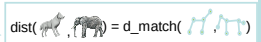
descriptions



histograms, matrices, graphs …

similarity measures

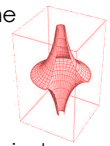
real numbers



metric
semi-metric
…
graph matching
….

things are not that easy…

- ✓ the simple examples we have shown are clearly not enough to deal with the *complexity* at a hand…
- ✓ we need tools to reason about
  - connectivity, interior, exterior and boundary
  - measuring shape properties and invariants
  - well-posedness
  - robustness and stability
  - distance and proximity
  - etc…



DON'T PANIC